Objectives:
- to write and apply exponential functions from two points
- to recognize an equation from a set of points
- create and solve doubling time and half life equations

1) Write an exponential function \( y = ab^x \) whose graph passes through \((1, 12)\) and \((3, 108)\).

Substitute \((x, y)\) into \(y = ab^x\)

\[
12 = ab^1 \quad \text{or} \quad a = \frac{12}{b}
\]

\[
108 = ab^3 \quad \text{or} \quad a = \frac{108}{b^3}
\]

Taking \(a = \frac{12}{b}\) and \(a = \frac{108}{b^3}\), we get

\[
\frac{12}{b} = \frac{108}{b^3}
\]

Solving for \(b\),

\[
b = \sqrt[3]{\frac{12}{108}} = \sqrt[3]{\frac{1}{9}} = \frac{1}{b}
\]

\[
12 = a \cdot \left(\frac{12}{b}\right)^3
\]

\[
108 = a \cdot \left(\frac{12}{b}\right)^2
\]

Solving for \(b\),

\[
b = \sqrt{\frac{12}{108}} = \sqrt{\frac{1}{9}} = \frac{1}{b}
\]

\[
12 = 4 \cdot 3^x
\]

Therefore, \(y = 4 \cdot 3^x\).
2) Write an exponential function $y = ab^x$ whose graph passes through (3, 0.5) and (-4, 64).

\[
\begin{align*}
(3, 0.5) & \quad y = ab^x \\
0.5 & = ab^3 \\
\frac{a}{b^3} & = 0.5
\end{align*}
\]

\[
\begin{align*}
(-4, 64) & \quad y = ab^x \\
64 & = ab^{-4} \\
64 & = \left(\frac{0.5}{b^3}\right)^{-4} \\
b^{-4} & = \frac{1}{b^4}
\end{align*}
\]

\[
\begin{align*}
b^{-4} & = \left(\frac{0.5}{b^3}\right)^{-4} \\
b & = \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
a & = 0.5 \\
a & = \frac{0.5}{b^3} \\
a & = \frac{0.5}{\left(\frac{1}{2}\right)^3} = \frac{\frac{1}{2}}{\frac{1}{8}} = 4 = a
\end{align*}
\]

\[
\begin{align*}
y & = 4 \cdot \left(\frac{1}{2}\right)^x
\end{align*}
\]
3) An online store begins selling a new type of basketball shoe. In week 2, 60 pairs of shoes were sold. In week 4, the store sold 240 pairs.

Write an exponential model \( y = ab^x \) that relates the number of shoes (in pairs) sold to the week number.

\[
(2, 60) \quad \quad \quad \quad 2^x \cdot \frac{60}{b^2} = \frac{60}{b^2} = 60 \cdot b^2
\]

\[
(4, 240) \quad \quad \quad \quad 4^x \cdot \frac{60}{b^2} = 240 \Rightarrow \frac{60}{b^2} = 60 \cdot b^2
\]

\[
\frac{60}{b^2} = \frac{60}{b^2} \Rightarrow 60 = 60 \cdot b^2
\]

\[
a = \frac{60}{b^2}
\]

\[
a = \frac{60}{4} = 15
\]

How many did they originally sell?

15 pairs = \( a \) value

What is the growth factor?

\( b = 2 \)

What is the % increase?

\[
2 = 1 + r \quad \quad \quad \quad 1 = r \quad \quad \quad \quad r = 100\%
\]

How many shoes (if the trend continues) will be sold in week 6?

\[
y = 15 \cdot 2^6
\]

\[
y = 15 \cdot 64
\]

\[
y = 960 \text{ pairs of shoes}
\]
Determine which ordered pairs are solutions to the given function.

4) \[ y = -3 \left( \frac{1}{2} \right)^x \]

A) \((0, -3)\)  \[ y = -3 \left( \frac{1}{2} \right)^0 = -3 \]

B) \((2, -0.75)\)  \[ y = -3 \left( \frac{1}{2} \right)^2 = -3 \cdot \frac{1}{4} = -\frac{3}{4} \]

C) \((-1, 6)\)  \[ y = -3 \left( \frac{1}{2} \right)^{-1} = -3 \cdot 2 = -6 \]

D) \((-2, -12)\)  \[ y = -3 \left( \frac{1}{2} \right)^{-2} = -3 \cdot 4 = -12 \]
5) For your 16th birthday, you received $250 from your grandparents. (You have very nice grandparents.) You plan to deposit this gift in your savings account that earns 5% interest compounded annually.

\[ y = a(1+r)^t \]

Write an equation in standard exponential form for the amount in your account after x years.

\[ y = \frac{250(1 + 0.05)^x}{250 (1.05)^x} \]

How much money will be in your account on your 18th birthday given no deposits or withdrawals are made?

\[ y = 250(1.05)^2 \]

\[ y = \$275.63 \]

How many years will it take to double your money?

\[ \frac{500}{250} = \frac{500}{250 (1.05)^x} \]

\[ x \approx 14.207 \text{ years} \]

In about 14.207 years, I would double my money.

If your goal is to save this money to purchase a used car and the car you have in mind costs $5210.99, how long will it take to have enough money to make the purchase?

\[ 5210.99 = 250(1.05)^{\frac{y}{2}} \]

\[ 62.247 \text{ yrs} \]
Doubling Time: \( y = a(2)^x \)  
Half Life: \( y = a\left(\frac{1}{2}\right)^h \)

- \( y \): final amount after "x" events
- \( a \): initial amount
- \( x \): # of time intervals
- \( d \): doubling time
- \( h \): half life
6) Suppose you have $475 invested in a bank. Your bank advisor tells you to expect your money to double in about twelve years.

Write an exponential equation to model the situation described.

\[ y = a(2)^{x/12} \]

\[ y = 475(2)^{x/12} \]

\[ x = \text{time in years} \quad y = \text{total amt. in account} \]

Find the balance of your account after 3.5 years.

\[ y = 475(2)^{3.5/12} \]

\[ y = 581.43 \]

:. I will have $581.43 after 3.5 years.

When could you expect your bank balance to be $1000 if neither deposits nor withdrawals are made?

\[ 1000 = 475(2)^{x/12} \]

\[ y_2 \]

\[ y_2 \]

\[ (12.888, 1000) \]

\[ (12.888, 1000) \]

\[ \text{Calc. Int.} \]

\[ 12.888 (\approx 13 \text{ years}) \]

\[ \text{It would take 12.888 (\approx 13 \text{ years}) years to have $1000 in the account.} \]
7) A doctor's office did an audit on its paper usage. In an effort to help the environment and reduce overhead cost, the office wants to cut the amount of paper usage in half over the next three years. They currently use 2250 reams of paper per year.

Write an exponential equation to model the situation described.

\[ y = a \left( \frac{1}{2} \right)^{x/3} \]

\[ y = 2250 \left( \frac{1}{2} \right)^{x/3} \]

\( x = \text{time in years} \)

\( y = \text{total amt. of paper} \)

How long will it take for the office staff to reduce their paper usage to 2000 reams per year?

\[ 2000 = 2250 \left( \frac{1}{2} \right)^{x/3} \]

In about 6 months, the office will be using 2000 reams.

If the audit was completed in 2013 and the company could realistically continue cutting their paper usage in half every three years, how much paper will they be using in 2021?

\[ y = 2250 \left( \frac{1}{2} \right)^{8/3} \]

\[ y = 2250 \left( \frac{1}{2} \right)^{8/3} \]

\[ y = 354.353 \]

In 8 years, the office will use about 354 reams.
Assignment:
Worksheet #3
$$y = 3^x$$

v. shift up 2 units
$$y = 3^x + 2$$

Roy (reflection across y-axis)
$$y = 3^{-x} + 2$$

RoX
$$y = -3^{-x} + 2$$
\[ y = \left( \frac{3}{2} \right)^x \]

1) Red \[ y = \left( \frac{3}{2} \right)^{-x} \]

2) Rox \[ y = -\left( \frac{3}{2} \right)^x \]

3) V. shift \[ \text{down 4 units} \] \[ y = -\left( \frac{3}{2} \right)^x - 4 \]
\[
13 \quad -5u^{20}\left(2^{1 \frac{1}{2}} - 3\right)^8 \\
-5u^{20}(16u^{-24}) \\
\]
\[
\begin{align*}
\text{\textcolor{red}{-80u^{-4}}} \\
\frac{-80}{u^4}
\end{align*}
\]
\[
\begin{align*}
20 & -24 \\
\Rightarrow & u \\
20 + (-24) = & -4
\end{align*}
\]
\[ 1.07 \]

\[ 1 + r \]

\[ 1.07 = 1 + \sqrt{r} \]

\[ 1.07 = r \]

\[ 0.07 = r \]

\[ \approx 7\% \]
HW: Finish Day 3 WS (ALL questions) AND Transformations WS

Calc. Part of Quiz Tomorrow!