Curriculum: AP Calculus AB-II

Curricular Unit: Limits of Indeterminate Form

Instructional Unit: A. L'Hôpital's Rule – evaluate limits using L'Hôpital's Rule

Description Section in Schoolnet:
I.B. Functions, Graphs, and Limits: Limits of Functions
1. An intuitive understanding of the limiting process
2. Calculating limits using algebra

I.C. Functions, Graphs, and Limits: Asymptotic and unbounded behavior
2. Describing asymptotic behavior in terms of limits involving infinity

II.F. Derivatives: Computation of Derivatives
1. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse functions
2. Derivative rules for sums, products, and quotients of functions
3. Chain rule and implicit differentiation

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2
CCSS: N/A
APCALC: AB.Ib.1,2; AB.Ic.2; AB.IIf.1-3
NETS: 1a; 6a
Performance: 1.6, 3.2

Unit (Section 3)

Learning Targets:

- Use L'Hôpital's Rule to evaluate limits of indeterminate forms:
  - $\left(0/0 \; and \; \infty/\infty\right)$
  - $\left(\infty-\infty, 0\cdot\infty, 0^0, 1^\infty, \infty^0\right)$

Instructional Strategies:

- Lecture enhanced with:
  - SMART Notebook
  - PowerPoint
  - the Internet
- Drill and guided practice
- Demonstrations
- Activity: Students will review L'Hôpital's Rule using dry erase boards
- Reflective discussion
- Class discussion
- Computer assisted instruction

Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
  - Direct teacher observations
  - Formative assessments
  - Homework assignments
  - Formal common assessment

Mastery: 80%

Sample Assessment Questions:

- $\lim_{x \to 0^+} (1 - 2x)^{\frac{1}{x}}$

Instructional Resources/Tools:

- Textbook(s): Chapter 6 - Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005.
- Website(s): http://www.math.oregonstate.edu/home/programs/undergrad/CalculusQuestStudyGuides/SandS/lHopital/ (1996), Khan Academy
- Graphing calculator

Cross Curricular Connections:

-

Depth of Knowledge (Section 5)

DOK: 4

Curriculum: AP Calculus AB-II

Curricular Unit: Differential Equations

Instructional Unit: B. The calculus of growth and decay – create and solve differential equations

Description Section in Schoolnet:
II.E. Derivatives: Applications of Derivatives

6. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

III.E. Integrals: Applications of Antidifferentiation

1. Finding specific antiderivatives using initial conditions, including applications to motion along a line
2. Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2
CCSS: N/A
APCALC: AB.IIe.6; AB.IIIe.1,2
NETS: 1a; 6a
Performance: 1.6, 1.10

Unit (Section 3)

Learning Targets:

- Solve variable separable differential equations

- Given a real-world situation in which the rate of change of y with respect to x is directly proportional to y, write and solve a differential equation and use the resulting solution as a mathematical model to make predictions and interpretations of that real-world situation

- Given the relationship between a function and its rate of change, write a differential equation, solve it to find an equation for the function, and use the function as a mathematical model

- Graph a particular solution for a differential equation by hand given a slope field

Instructional Strategies:

- Lecture enhanced with:
  - SMART Notebook
  - PowerPoint

- the Internet
- Demonstrations
- Activity: Wiki Stick Slope Fields: Create slope fields for several differential equations by placing wiki sticks on laminated slope field graphs
- Reflective discussion
- Class discussion
- Drill and guided practice
- Computer assisted instruction

Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
  - Direct teacher observations
  - Formative assessments
  - Homework assignments
  - Formal common assessment

Mastery: 80%

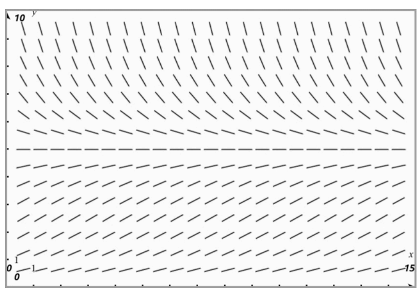
Sample Assessment Questions:

- Given the differential equation: $\frac{dy}{dx} = .45y - .09y^2$

  A) Find the slope of the particular equation at each of the following points:
  i) (3, 4) ii) (8, 5) iii) (10, 8)

  B) Sketch the particular solution through each of the points in part A) Make each a different color.

  C) What are the major similarities and differences between each of the solutions?



Instructional Resources/Tools:

- Textbook(s): Chapter 7 – Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005.
- Website(s): https://sites.google.com/site/sanfordmartind/ap-calculus-slope-fields (1996), Khan Academy
- Graphing calculator

Cross Curricular Connections:

-

Depth of Knowledge (Section 5)

DOK: 4

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Curriculum: AP Calculus AB-II

Curricular Unit: Critical Points and Points of Inflection

Instructional Unit: C. Functions and their derivatives – analyze the graphs of functions, derivatives, and antiderivatives

Description Section in Schoolnet:

I.A. Functions, Graphs, and Limits: Analysis of Graphs

1. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function

II.C. Derivatives: Derivative as a Function

1. Corresponding characteristics of graphs of f and f'
2. Relationships between the increasing and decreasing behavior of f and the sign of f'

II.D. Derivatives: Second Derivatives

1. Corresponding characteristics of the graphs of f, f', f''
2. Relationships between the concavity of f and the sign of f'
3. Points of inflection as places where concavity changes

II.E. Derivatives: Applications of Derivatives

1. Analysis of curves, including the notions of monotonicity and concavity

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2,3
CCSS: N/A
APCALC: AB.Ia.1; AB.IIc.1,2; AB.IId.1-3; AB.IIe.1
NETS: 1a; 6a
Performance: 1.6, 4.1

Unit (Section 3)

Learning Targets:

- Find critical points, extrema, and points of inflection algebraically, graphically, and numerically
- Draw the graph of a derivative or antiderivative of a function given its graph
- Find global (absolute) extrema given an equation and a domain

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Instructional Strategies:

- Lecture enhanced with:
    - SMART Notebook
    - PowerPoint
    - the Internet
- Demonstrations
- Activity: The Great Matching Caper: Students will match graphs of *f, f', f''* and their descriptions to each other in collaborative groups
- Reflective discussion
- Class discussion
- Drill and guided practice
- Computer assisted instruction

Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
    - Direct teacher observations
    - Formative assessments
    - Homework assignments
    - Formal common assessment

Sample Assessment Questions:

- Sketch a graph that is continuous on the interval from 1 to 7 such that $f(1)=0$ and $f(2)=1$ and that has the following properties:

	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$	4	$4<x<6$
f'	e.p.	+	∞	–	0	–	∞	+
f''	e.p.	+	∞	+	0	–	∞	–

Instructional Resources/Tools:

- Textbook(s): Chapter 8 - Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005.
- Website(s): https://sites.google.com/site/sanfordmartind/ap-calculus-slope-fields
- Graphing calculator

Cross Curricular Connections:

-

Depth of Knowledge (Section 5)

DOK: 4

Curriculum: AP Calculus AB-II

Curricular Unit: Maxima and Minima in Plane and Solid Figures

Instructional Unit: D. Optimization – solve optimization problems

Description Section in Schoolnet:
II.D. Derivatives: Second Derivatives
2. Relationships between the concavity of *f* and the sign of *f"*
II.E. Derivatives: Applications of Derivatives
2. Optimization, both absolute (global) and relative (local) extrema

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2
CCSS: N/A
APCALC: AB.IId.2; AB.IIe.2
NETS: 1a; 6a
Performance: 1.8, 3.1

Unit (Section 3)

Learning Targets:

- Solve optimization problems involving plane or solid figures (area, perimeter, volume)

Instructional Strategies:

- Lecture enhanced with:
    - SMART Notebook
    - PowerPoint
    - the Internet
- Demonstrations
- Activity: Open Box – Students will find dimensions to maximize volume given a specific grid size
- Reflective discussion
- Class discussion
- Drill and guided practice
- Computer assisted instruction

Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
    - Direct teacher observations
    - Formative assessments
    - Homework assignments
    - Formal common assessment

Mastery Level: 80%

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Sample Assessment Questions:
• A piranha tank for the St. Louis Aquarium is to be constructed to hold 70 cubic feet of water. Its base and sides are to be rectangular. The top, of course, is to be open. The width is 5 feet, but the length and height are variable. Building the tanks costs \$15.75 per square foot for the sides, and \$20.50 per square foot for the base. Find the dimensions of the tank that will minimize the cost. Find that cost
Instructional Resources/Tools: • Textbook(s): Chapter 8 – Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005. • Website(s): http://tutorial.math.lamar.edu/Classes/CalcI/Optimization.aspx • Graphing calculator
Cross Curricular Connections: •

Depth of Knowledge (Section 5)

DOK: 4

Board Approved 8-3-15

Curriculum: AP Calculus AB-II

Curricular Unit: Volume, Area and Arc Length of Plane and Solid Figures

Instructional Unit: E. The calculus of plane and solid figures – evaluate integrals used to find area, volume, arc length, and surface area

Description Section in Schoolnet:

III.B. Integrals: Applications of Integrals

1. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Specific applications should include finding the area of a region, the volume of a solid with known cross sections, and the length of a curve

III.C. Integrals: Fundamental Theorem of Calculus

1. Use the Fundamental Theorem of Calculus to evaluate definite integrals

Standard Alignments (Section 2)

GLE/CLE: N/A Knowledge: (MA) 2,4 CCSS: N/A APCALC: AB.IIIb.1; AB.IIIc.1 NETS: 1a,c; 4b; 6a Performance: 1.6, 3.1

Unit (Section 3)

Learning Targets: • Find the area of a planar region • Find volumes of solids of revolution (methods: disc, washer or ring, shells) • Find volumes of solids with known cross sections • Find the arc length of a curve • Find the surface area of a solid of revolution

Instructional Strategies:
 	
• Lecture enhanced with:
    • SMART Notebook
    • PowerPoint
    • the Internet
• Drill and guided practice
• Demonstrations

- Activity:
  - The Dam Project – Students will calculate cost involved in building a dam when given certain restrictions
  - Models of Cardboard and Play-Doh – Students will build models and use them to find volumes
- Reflective discussion
- Class discussion
- Computer assisted instruction

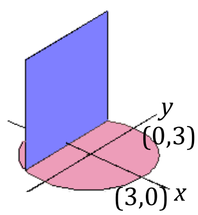
Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
  - Direct teacher observations
  - Formative assessments
  - Homework assignments
  - Group project
  - Formal common assessment

Mastery Level: 80%

Sample Assessment Questions:

- Find the volume of a solid whose base is bounded by the circle $x^2 + y^2 = 9$ and cross-sections perpendicular to the x-axis are squares



Instructional Resources/Tools:

- Textbook(s): Chapter 8 – Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005.
- Website(s):
  - http://web.monroecc.edu/manila/webfiles/pseeburger/secure/MyLarson/LarsonApplets.htm
  - Dam Project Song: https://www.youtube.com/watch?v=5vLZOKshJPs
- Graphing calculator

Cross Curricular Connections:

-

Depth of Knowledge (Section 5)

DOK: 4

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Curriculum: AP Calculus AB-II

Curricular Unit: Techniques of Integration

Instructional Unit: F. Algebraic calculus techniques for the elementary functions – apply various techniques of integration to evaluate a given integral

Description Section in Schoolnet:
III.D. Integrals: Techniques of Antidifferentiation

1. Antiderivatives following directly from derivatives of basic functions
2. Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, simple partial fractions

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2
CCSS: N/A
APCALC: AB.IIId.1,2
NETS: 1a; 6a
Performance: 1.8, 3.1

Unit (Section 3)

Learning Targets:

- Use integration by parts to find the integral of a product
- Use reduction formulas to evaluate an indefinite integral
- Integrate using odd/even powers of sine/cosine, even powers of secant/cosecant, inverse trig functions
- Use trigonometric substitution to integrate
- Use partial fractions to integrate

Instructional Strategies:

- Lecture enhanced with:
  - SMART Notebook
  - PowerPoint
  - the Internet
- Demonstrations
- Activity: Trig Substitution – Students will practice setting up triangles and using them to write a trigonometric equation, which will later be used for various integrands

• Reflective discussion • Class discussion • Drill and guided practice • Computer assisted instruction

Assessments/Evaluations:
 	
• The students will be assessed on the concepts taught using a variety of modalities:
    • Direct teacher observations
    • Formative assessments
    • Homework assignments
    • Formal common assessment
 	
Mastery Level: 80%

Sample Assessment Questions: • Find the exact volume of when the region *R* under $y = \cos x$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ is revolved around the ***x-axis*** • Draw graphs, label, etc.
Instructional Resources/Tools: • Textbook(s): Chapter 9 – Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005. • Graphing calculator
Cross Curricular Connections: •

Depth of Knowledge (Section 5)

DOK: 4

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Curriculum: AP Calculus AB-II

Curricular Unit: Distance, Displacement, and Acceleration; Average Value; Minimal Path

Instructional Unit: G. The calculation of motion – solve problems involving motion and related rates

Description Section in Schoolnet:

II.A. Derivatives: Concept of the Derivative

2. Derivative interpreted as an instantaneous rate of change

II.E. Derivatives: Applications of Derivatives

5. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

II.F. Derivatives: Computation of Derivatives

1. Knowledge of derivatives of basic functions

III.A. Integrals: Interpretations and Properties of Definite Integrals

2. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval

III.B. Integrals: Applications of Integrals

1. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Specific applications should include the average value of a function

III.E. Integrals: Applications of Antidifferentiation

1. Finding specific antiderivatives using initial conditions, including applications to motion along a line

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2
CCSS: N/A
APCALC: AB.IIa.2; AB.IIe.5; AB.IIf.1; AB.IIIa.2; AB.IIIb.1; AB.IIIe.1
NETS: 1a; 6a
Performance: 1.8, 3.1

Unit (Section 3)

Learning Targets:

- Find the displacement at a given time and the distance traveled in a given time interval given velocity or acceleration as a function of time for an object in linear motion
- Calculate the average value of a function
- Predict the rate at which a quantity is changing given the other necessary information

- Use derivatives to minimize distance, time, or cost
- Find maximums and minimums for real world situations

Instructional Strategies:

- Lecture enhanced with:
  - SMART Notebook
  - PowerPoint
  - the Internet
- Demonstrations
- Activity: Exploration 10 – 3 – Students will explore the solution to a problem related to average velocity
- Reflective discussion
- Class discussion
- Drill and guided practice
- Computer assisted instruction

Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
  - Direct teacher observations
  - Formative assessments
  - Homework assignments
  - Formal common assessment

Mastery Level: 80%

Sample Assessment Questions:

- A ladder 13 feet long is leaning against the side of a building. If the foot of the ladder is pulled away from the building (making the top of the ladder slide down the building) at a constant rate of 6 inches per second. When the bottom of the ladder is 12 feet from the wall, how fast is the top of the ladder approaching the ground?

Instructional Resources/Tools:

- Textbook(s): Chapter 10 – Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005.
- Website(s):
  - http://faculty.bucks.edu/taylors/calculus/ProbsRelatedRates.pdf
  - http://www.maitespace.com/math/Calculus/APUnit3/RelRates.htm
- Graphing calculator

Cross Curricular Connections:

-

Depth of Knowledge (Section 5)

DOK: 4

Curriculum: AP Calculus AB-II

Curricular Unit: Hyperbolic Function

Instructional Unit: H. Calculus of the hyperbolic functions – investigate hyperbolic trig functions and apply calculus to them

Description Section in Schoolnet:

I.A. Functions, Graphs, and Limits: Analysis of Graphs

1. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function

II.A. Derivatives: Concept of the Derivative

1. Derivative presented graphically, numerically, and analytically

III.A. Integrals: Interpretations and Properties of Definite Integrals

3. Basic properties of definite integrals

III.B. Integrals: Applications of Integrals

1. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Specific applications should include the length of a curve

Standard Alignments (Section 2)

GLE/CLE: N/A
Knowledge: (MA) 2,4
CCSS: N/A
APCALC: AB.Ia.1; AB.IIa.1; AB.IIIb.3; AB.IIIb.1
NETS: 1a,c; 4c
Performance: 3.2

Unit (Section 3)

Learning Targets:

- Graph equations relating to hyperbolic functions
- Find derivatives of equations involving hyperbolic functions
- Find integrals of equations involving hyperbolic functions
- Solve application problems involving hyperbolic functions

Instructional Strategies:

- Lecture enhanced with:
  - SMART Notebook
  - PowerPoint
  - the Internet
- Demonstrations
- Activity: Hanging Chain Lab – use a coordinate system with a hanging chain to find the equation of the catenary curve and use the equation to find the length of the chain
- Reflective discussion
- Class discussion
- Drill and guided practice
- Computer assisted instruction

Assessments/Evaluations:

- The students will be assessed on the concepts taught using a variety of modalities:
  - Direct teacher observations
  - Quiz
  - Homework assignments
  - Formal common assessment

Mastery Level: 80%

Sample Assessment Questions:

- Missouri S&T engineering students love to build scaled down models of real things, e.g., Stonehenge. They are also planning on building a model of the St. Louis Arch. The outside of this model arch is 157.5 feet wide at the base and 157.5 feet high. The inside of the arch at the base is 130 feet wide and 153 feet high.
  A) Find particular equations for the inside and outside catenary curves.
  B) Graph and label each equation from part A.
  C) The stress created by wind blowing against the arch depends on the area of the region between the two graphs. Find this area.
  D) A centipede starts at the point where the left end of the outside arch meets the ground and crawls all the way up, then down the other side to the ground. How far did it crawl?

Instructional Resources/Tools:

- Textbook(s): Chapter 9 - Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. California: Key Curriculum Press, 2005.
- Graphing calculator

Cross Curricular Connections:

-

Depth of Knowledge (Section 5)

DOK: 4

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